THE ALGEBRA UNIT — EVERYTHING IN ONE PLACE

Polynomials & Algebraic Identities

Every formula you need for the algebra portion — polynomials, algebraic identities, factorisation, linear equations in two variables, and the quadratic-form identities — laid out chapter by chapter for quick revision.

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WHAT'S INSIDE — 5 SECTIONS

THE FIVE IDENTITIES YOU MUST NEVER FORGET

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$a^2 - b^2 = (a + b)(a - b)$

$(x + a)(x + b) = x^2 + (a+b)x + ab$

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

1 Polynomials

Types · degree · zeroes · remainder & factor theorem

Basic definitions

Polynomial: $p(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$ where the powers of x are whole numbers and $a_n \neq 0$.

Degree = highest power of the variable

Degree	Name	Example
0	Constant	7
1	Linear	$2x + 3$
2	Quadratic	$x^2 - 5x + 6$
3	Cubic	$x^3 + 1$

By number of terms

Terms	Name	Example
1	Monomial	$5x^2$
2	Binomial	$x + 2$
3	Trinomial	$x^2 + x + 1$

A **zero polynomial** is 0; its degree is **not defined**. The degree of a non-zero constant is **0**.

Value & zero of a polynomial

Value: $p(a)$ — substitute $x = a$ into $p(x)$.

Zero / root: a number k with $\mathbf{p(k) = 0}$.

- Zero of a **linear** polynomial $ax + b$ is $\mathbf{x = -b/a}$.
- A polynomial of degree n has **at most n** zeroes.
- Geometrically, a zero is where the graph **cuts the x-axis**.

Remainder theorem & factor theorem

If $p(x)$ is divided by $(x - a)$, the remainder is $p(a)$.

Factor theorem: $(x - a)$ is a factor of $p(x)$ $\iff$ $\mathbf{p(a) = 0}$.

Also: divide by $(x + a)$ → remainder is $\mathbf{p(-a)}$; by $(ax - b)$ → remainder $p(b/a)$.

HOW TO USE THEM To check a factor, put the divisor = 0, solve for x, and substitute into $p(x)$. If the result is **0**, it is a factor. This is the key to factorising **cubic** polynomials — find one zero by trial (from the factors of the constant term), then divide.

2 Algebraic Identities

Squares · products · cubes · the three-variable identity

Square identities

$(a+b)^2 = a^2 + 2ab + b^2$

$a^2 + b^2 = (a+b)^2 - 2ab$

$(a-b)^2 = a^2 - 2ab + b^2$

$a^2 + b^2 = (a-b)^2 + 2ab$

$a^2 - b^2 = (a+b)(a-b)$

$(a+b)^2 - (a-b)^2 = 4ab$
and $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$

$(x+a)(x+b) = x^2 + (a+b)x + ab$ — the product identity used to split the middle term.

Three-variable square identity

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$\Rightarrow a^2 + b^2 + c^2 = (a+b+c)^2 - 2(ab+bc+ca)$

Cube identities

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
$= a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
$= a^3 - 3a^2b + 3ab^2 - b^3$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$$a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

SPECIAL CASE If $a+b+c=0$, then $a^3+b^3+c^3=3abc$. This shortcut appears very often in exams.

3 Factorisation & Linear Equations

Methods of factorising · lines in two variables

Methods of factorisation

1 · Taking out the common factor

$ab + ac = \mathbf{a(b + c)}$ · $6x^2 + 9x = \mathbf{3x(2x + 3)}$

2 · Grouping the terms

$ax + ay + bx + by = a(x + y) + b(x + y) = \mathbf{(x + y)(a + b)}$

3 · Splitting the middle term — for $x^2 + bx + c$

Find two numbers whose **sum = b** (middle coefficient) and **product = c** (last term). For $ax^2 + bx + c$, use product = $\mathbf{a \times c}$.

e.g. $x^2 + 7x + 12 \to 3 + 4 = 7$, $3 \times 4 = 12 \to (x + 3)(x + 4)$

4 · Using an identity

$a^2 - b^2 \to \mathbf{(a + b)(a - b)}$
$a^2 \pm 2ab + b^2 \to \mathbf{(a \pm b)^2}$

$a^3 \pm b^3 \to \mathbf{(a \pm b)(a^2 \mp ab + b^2)}$
Cubics → find one zero, then divide (factor theorem)

Linear equations in two variables

Standard form $ax + by + c = 0$ (a, b not both zero)

- A **solution** is a pair (x, y) that satisfies the equation.
- Every such equation has **infinitely many** solutions.
- Its graph is always a **straight line**.
- Every point on the line is a solution, and every solution lies on the line.

$\mathbf{y = mx + c}$ slope–intercept form; m = slope, c = y-intercept

$\mathbf{x = a}$ → line **parallel to the y-axis**
$\mathbf{y = b}$ → line **parallel to the x-axis**

A line through the origin has form $\mathbf{y = mx}$ $(c = 0)$.

Useful transformations

$\mathbf{a^2 + 1/a^2}$
$= (a + 1/a)^2 - 2$

$\mathbf{a^2 + 1/a^2}$
$= (a - 1/a)^2 + 2$

$\mathbf{a^3 + 1/a^3}$
$= (a + 1/a)^3 - 3(a + 1/a)$

REMEMBER These "reciprocal" forms come straight from the square and cube identities with $b = 1/a$. They are common in "find the value" problems where $a + 1/a$ is given.

4 Quadratic Form & Quick Reference

The quadratic identities · one-look revision table

Quadratic-form identities

$ax^2 + bx + c$

standard quadratic polynomial ($a \neq 0$)

$x^2 + (a + b)x + ab = (x + a)(x + b)$

NOTE FOR CLASS 9 In Class 9 a quadratic is **factorised** by splitting the middle term or using identities. Solving with the **quadratic formula** $x = [\,-b \pm \sqrt{(b^2 - 4ac)}\,] / 2a$ and the **discriminant** $D = b^2 - 4ac$ belong to **Class 10** — shown here only so the set is complete.

$$x = \left(-b \pm \sqrt{(b^2 - 4ac)} \right) / 2a \quad \cdot \quad D = b^2 - 4ac$$ (Class 10 preview)

One-look revision table

Identity	Expansion / Factor form
$(a + b)^2$	$a^2 + 2ab + b^2$
$(a - b)^2$	$a^2 - 2ab + b^2$
$a^2 - b^2$	$(a + b)(a - b)$
$(x + a)(x + b)$	$x^2 + (a + b)x + ab$
$(a + b + c)^2$	$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
$(a + b)^3$	$a^3 + b^3 + 3ab(a + b)$
$(a - b)^3$	$a^3 - b^3 - 3ab(a - b)$
$a^3 + b^3$	$(a + b)(a^2 - ab + b^2)$
$a^3 - b^3$	$(a - b)(a^2 + ab + b^2)$
$a^3 + b^3 + c^3 - 3abc$	$(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
If $a + b + c = 0$	$a^3 + b^3 + c^3 = 3abc$

Exam pointers & common mistakes

- $(a + b)^2$ is **not** $a^2 + b^2$ — never drop the **2ab**.
- $a^2 - b^2$ factorises; **$a^2 + b^2$** does **not** (over real numbers).
- In splitting the middle term for $ax^2 + bx + c$, use product **$a \times c$**, not just c.
- Remainder when dividing by $(x + a)$ is **$p(-a)$**, not $p(a)$.
- Degree of the **zero polynomial** is not defined.
- Watch the signs in $a^3 - b^3$ vs $(a - b)^3$ — they are different.

MEMORY HOOKS ▸ Square of a sum = **square, square, twice the product**. ▸ Sum of cubes keeps a **minus** in the middle: $(a+b)(a^2-ab+b^2)$. ▸ When the three parts add to zero, cubes collapse to **3abc**. ▸ Factor theorem: **$p(a)=0 \Longrightarrow (x-a)$ is a factor**.